

Use of truss

- └─ Roof truss
- └─ Bridge truss

Assumptions involved in the analysis of truss-

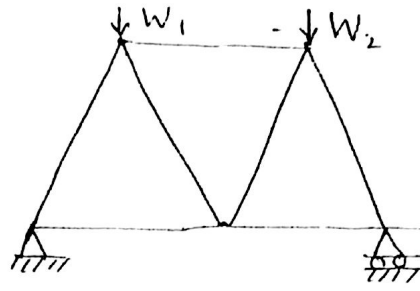
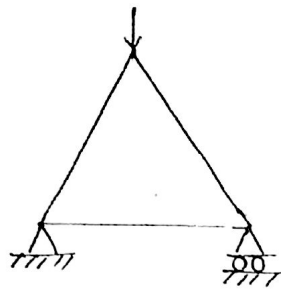
- (1). The material is isotropic, homogeneous and linear elastic.
- (2). All joints are pin connected i.e. hinged, and all hinges are frictionless.
- (3). Self weight of member is ignored.
- (4). Loading is applied only at joints which may be horizontal vertical @ inclined.
- (5). Members carry only axial forces (compression @ tension) and are free from shear force and B.M.

Types of truss on the basis of their structural arrangement

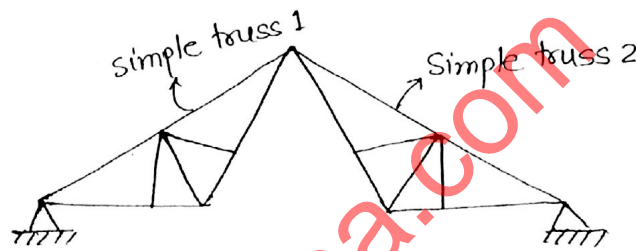
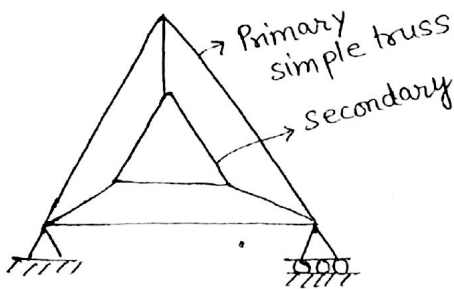
Plane trusses (coplanar trusses) are classified as-

- (1). Simple truss
- (2). Compound truss
- (3). Complex truss

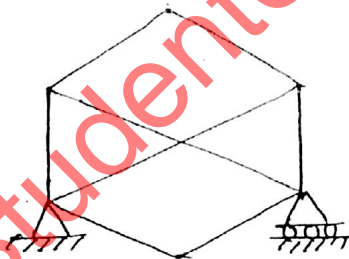
- (1). Simple Truss- A simple truss has a stable frame work consisting triangular arrangement of members. The simplest truss is a triangle consisting three members.



2). Compound Truss- Compound truss is formed by connecting two or more simple trusses together. It is often used for longer span and is proved cheaper than a simple truss for same span.



3). Complex Truss- If a truss is neither simple nor compound, then it will be called a complex truss. Generally a complex truss has polygonal structure.



Methods of Analysis of trusses (Plane trusses)

Determinate trusses
 $D_s = 0$

- (1). Method of Joint
- (2). Method of Section
- (3). Graphical Method

Indeterminate trusses
 $D_s > 0$ [$D_s = 1$]

- (1). Force method / strain energy method / unit load method / Maxwell Method.
- (2). Stiffness method / displacement method / equilibrium method
- (3). Graphical method.

Generally in trusses $D_s < D_k$ (much lower).
Hence force method are preferred.

Procedure of analysis of determinate trusses

Step 1 → Check for D_s . If $D_s = 0$, then truss is determinate and stable, such trusses are called pin jointed perfect frame. If $D_s > 0$ then truss is called redundant or over stiff and if $D_s < 0$ truss is called deficient and is unstable.

If $D_s = 0$ then find support reactions using conditions of static equilibrium.

- 1). $\sum F_x = 0$
- 2). $\sum F_y = 0$
- 3). $\sum M_z = 0$

These conditions are applied for the truss as a whole
Step 2 → To find member forces (axial force) either method of joint or method of section can be used

Method of Joint

This method is suitable when forces in all member of truss are required. This method is not applicable when no. of unknown at a joint are more than two because at each joint there are only two equations of equilibrium.

$$\sum F_x = 0 \quad ; \quad \sum F_y = 0$$

In method of joint the selection of joint is very judicious proceeding one by one such that no. of unknown forces at a joint are not more than two.

Method of Section

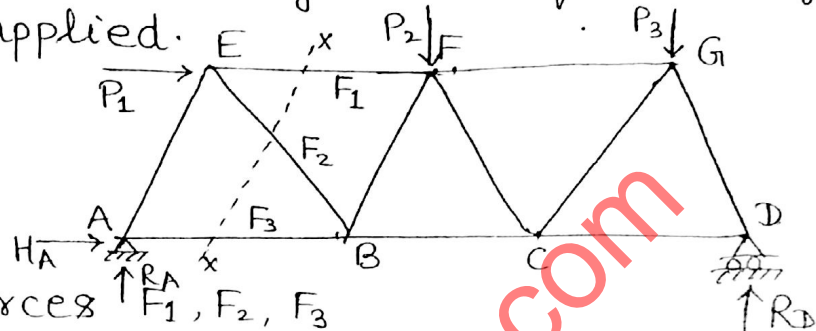
It is required when forces are required only in few members. After computation of support reaction, to determine member forces an imaginary section is cut such that not more than three unknown/members

should be present across the section. The section must cut those member in which forces are desired. The section cut may be horizontal / vertical @ zig-zag. To find forces in members free body equilibrium is established either to the left part of section x-x @ to the right part of section x-x and following three equations of equilibrium are applied.

$$1). \sum F_x = 0$$

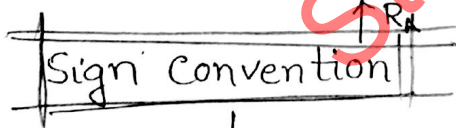
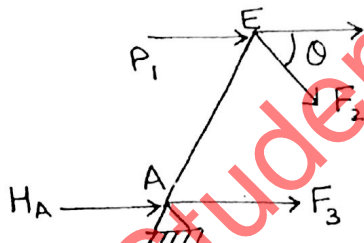
$$2). \sum F_y = 0$$

$$3). \sum M_z = 0$$

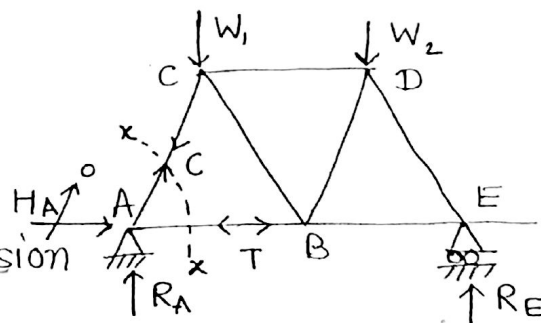


Let us find forces F_1, F_2, F_3

in members EF, EB and AB respectively. cut the section x-x such that section passes through these member and establish Free Body equilibrium either to the left part of x-x @ to the right part of x-x and apply above three equations of equilibrium in free body diagram to find F_1, F_2 and F_3 .

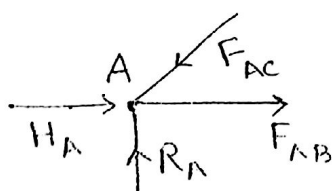


Axial Tension (+ve)
Axial Compression (-ve)



consider Joint A,

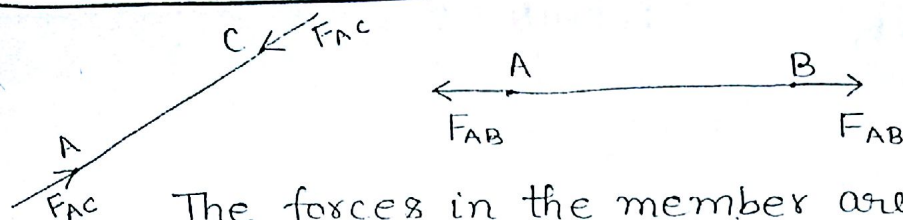
Let force in AB is tensile and in AC, it is compressive.



At a joint compression points towards the joint & Tension points away from joints.

$R_A \rightarrow$ compressive rxn. at A

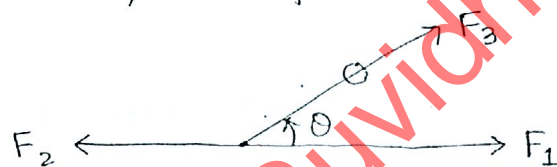
Member equilibrium



The forces in the member are shown by two arrows where as forces at joint are shown by single arrow. In members forces pointing away from each other are tensile and pointing towards each other are compressive.

Thumb rule to find member carrying zero forces

Thumb rule no. 1 → If at a joint three members meet and two of them are collinear and there is no external force @ reaction at that joint then third member will carry zero force.

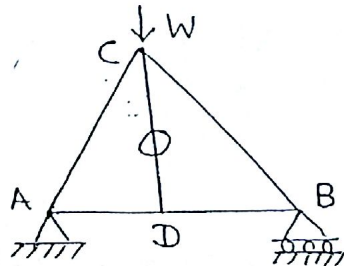


$$\sum F_y = 0 \Rightarrow F_3 \sin \theta = 0 \Rightarrow \boxed{F_3 = 0}$$

Note that $\sum F_x = 0$ then $F_1 - F_2 + F_3 \cos \theta = 0$

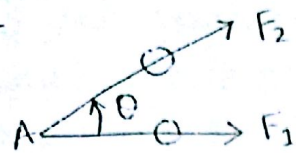
$$\boxed{F_1 = F_2}$$

It means if at joint two collinear members meet and there is no external force @ the reaction at that joint then both collinear member will have equal and alike forces i.e. either both in tension / compression @ zero.



Member CD has zero force.

Thumb rule no. 2 → If at a joint only two members meet which are non-collinear and there is no external force @ reaction at that joint then both members will carry zero force.



$$\theta \neq 0^\circ$$

$$\theta \neq 180^\circ$$

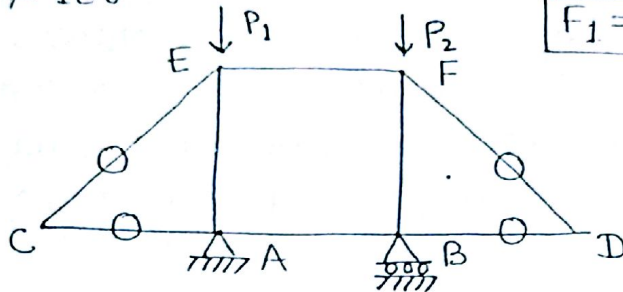
$$\sum F_y = 0 \text{ at joint A}$$

$$F_2 \sin \theta = 0$$

$$F_2 = 0$$

$$F_1 + F_2 \cos \theta = 0$$

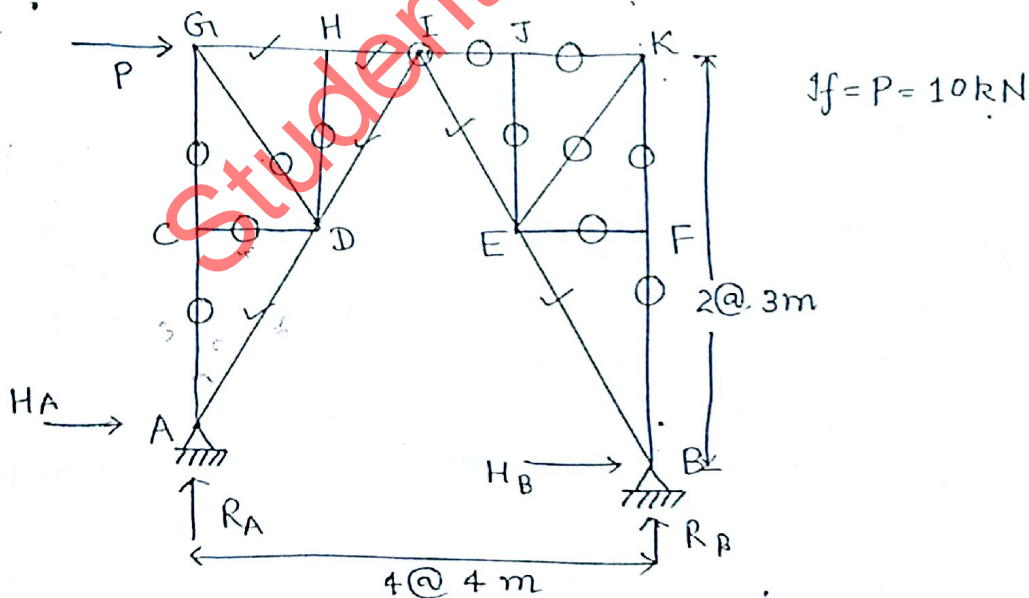
$$F_1 = 0$$



NOTE- If a truss under a specified loading carry some members with zero forces then such members should not be removed under changed loading condition such member may carry load. Due to removal of members frame (or) truss may become deficient and geometrically unstable.

Question Find no. of members with null forces in the truss shown in figure.

- (a) - 4 (b) - 8 (c) - 12 (d) - None of these.



$$D_s = m + R - 2j$$

$$= 18 + 4 - 2 \times 11 = 0$$

$$D_{se} = 1, D_{si} = -1$$

$$\sum F_x = 0,$$

$$H_A + H_B = -10 \quad \text{--- (1)}$$

$$\sum F_y = 0, R_A = -R_B = -3.75 \text{ kN}, \sum M_B = 0 \Rightarrow R_A = -3.75 \text{ kN}$$

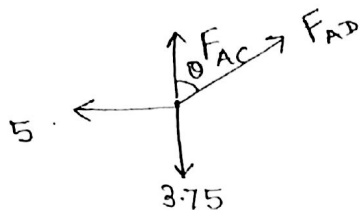
$$\sum M_I = 0$$

$$R_A \times 8 - H_A \times 6 = 0$$

$$H_A = -5 \text{ kN}$$

$$H_B = -5 \text{ kN}$$

consider joint A,



$$\sum F_x = 0,$$

$$F_{AD} \sin \theta = 5$$

$$F_{AD} = \frac{5 \times 5}{4} = 6.25$$

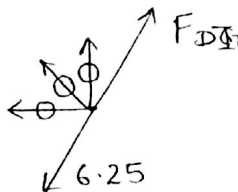
$$\sum F_y = 0,$$

$$F_{AD} \cos \theta + F_{AC} = 3.75$$

$$6.25 \times \frac{3}{5} + F_{AC} = 3.75$$

$$F_{AC} = 0$$

consider joint D;



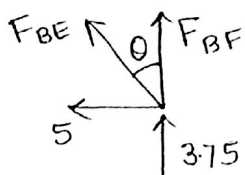
$$F_{DI} = 6.25$$

consider joint G,



$$F_{GH} = -10 \text{ kN}$$

consider joint B,



$$F_{BE} = -\frac{5}{\sin \theta} = -6.25$$

$$F_{BF} = -3.75 - F_{BE} \cos \theta = 0$$

Member

Force

(+) → Tension

(-) → compression

AC, CD, CE, DE, DH, BF 0

IJ, JK, EJ, EK, EF, KF 0

DI, AD

6.25 kN

BE

-6.25 kN

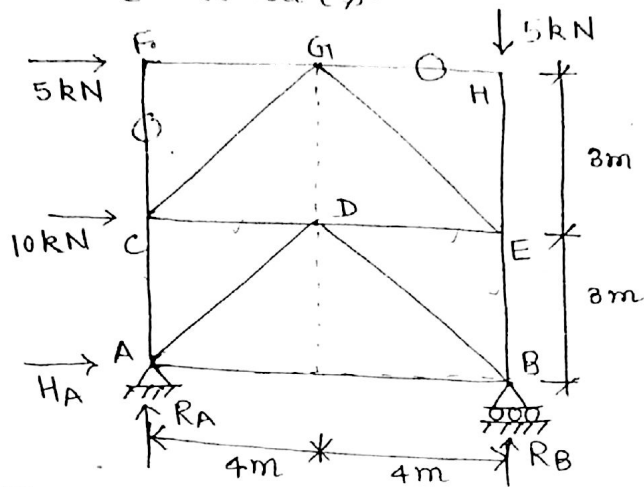
IE

-6.25 kN

GH, HI

-10 kN

Question For the truss shown in figure, find nature and magnitude of forces in the members. Tabulate the results.



$$D_s = m + R_e - 2j$$

$$= 13 + 3 - 2 \times 8$$

$$= 0$$

$$\sum F_y = 0,$$

$$R_A + R_B = 5$$

$$\sum F_x = 0,$$

$$H_A = -15$$

$$\sum M_A = 0,$$

$$R_B \times 8 = 40 + 30 + 30$$

$$R_B = \frac{100}{8} = 12.5 \text{ kN}$$

$$R_A = -1.5 \text{ kN}$$

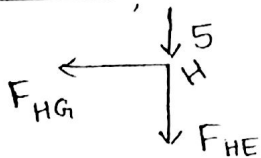
Joint F



$$F_{FG} = -5 \text{ kN}$$

$$F_{FC} = 0$$

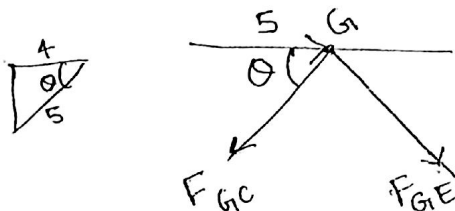
Joint H



$$F_{HE} = -5 \text{ kN}$$

$$F_{HG} = 0$$

Joint G



$$F_{GC} \cos \theta = 5 + F_{GE} \cos \theta \quad (1)$$

$$F_{GC} \sin \theta + F_{GE} \sin \theta = 0$$

$$\sin \theta (F_{GC} + F_{GE}) = 0$$

$$F_{GC} = -F_{GE}$$

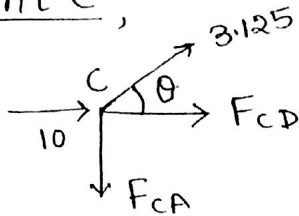
$$-F_{GE} \cos \theta \times 2 = 5$$

$$F_{GE} \times \frac{4}{5} \times 2 = -5$$

$$F_{GE} = \frac{-25}{8} = -3.125$$

$$F_{GC} = +3.125$$

Joint C,



$$F_{CD} + 3.125 \cos \theta = 10$$

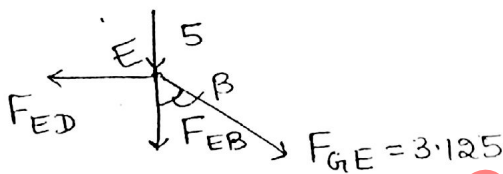
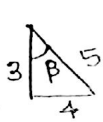
$$F_{CD} = -3.125 \times \frac{4}{5} - 10$$

$$= -12.5 \text{ kN}$$

$$F_{CA} = 3.125 \sin \theta$$

$$= 3.125 \times \frac{3}{5} = 1.875 \text{ kN}$$

Joint E,



$$5 + F_{EB} + 3.125 \cos \beta = 0$$

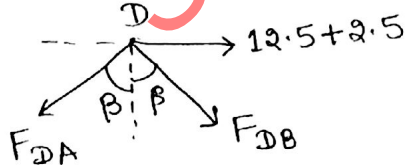
$$F_{EB} = -5 - 3.125 \times \frac{3}{5}$$

$$= -6.875$$

$$F_{ED} = 3.125 \sin \beta$$

$$= 3.125 \times \frac{4}{5} = 2.5$$

Joint D



$$F_{DB} \sin \beta + F_{DA} \sin \beta + 15 = 0$$

$$F_{DB} \cos \beta + F_{DA} \cos \beta = 0$$

$$F_{DB} = -F_{DA}$$

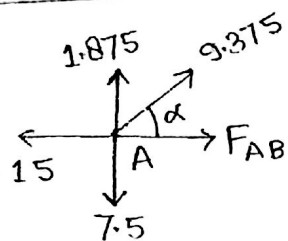
$$2F_{DB} \sin \beta = -15$$

$$2F_{DB} \times \frac{4}{5} = -15$$

$$F_{DB} = -9.375$$

$$F_{DA} = 9.375$$

Joint A,



$$F_{AB} + 9.375 \cos \alpha - 15 = 0$$

$$F_{AB} = 15 - 9.375 \times \frac{4}{5}$$

$$= 7.5 \text{ kN}$$

Check,

$$1.875 + 9.375 \sin \alpha = 7.5$$

$$1.875 + 5.625 = 7.5$$

$$7.5 = 7.5 \text{ OK}$$

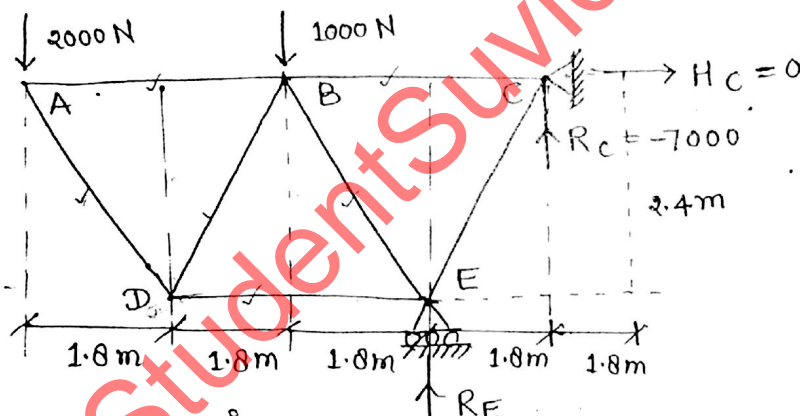
Member

Force

AB	+7.5
AC	+1.875
CF	0
CD	-12.5
AD	+9.375
BD	-9.375
BE	-6.875
CG	+3.125
DE	+2.5
EG	-3.125
EH	-5
FG	-5
GH	0

(Method of Tension coefficient)

Question]- Analyse the truss shown in figure and tabulate the forces in the member.



$$D_s = m + r_e - 2j$$

$$= 7 + 3 - 2 \times 5 = 0 \quad \checkmark \text{ Determinate}$$

$$\sum F_x = 0,$$

$$H_C = 0$$

$$\sum M_C = 0,$$

$$R_E \times 1.8 = 1000 \times 3.6 + 2000 \times 7.2$$

$$\sum F_y = 0,$$

$$R_E = 2000 + 8000$$

$$R_E + R_C = 3000$$

$$= 10,000 \text{ N}$$

$$\therefore R_C = -7000 \text{ N}$$

Considering joint D as origin, then the co-ordinates of different joints are,

	X	Y		
A	-1.8	2.4	AB	$t_{AB} \times \left(\frac{1000}{2.4}\right) = 1500$
B	1.8	2.4	AD	$t_{AD} \times \left(\frac{-2000}{2.4}\right) = -2500$
C	5.4	2.4	BE	$t_{BE} \times \left(\frac{3000}{2.4}\right) = -3750$
D	0	0	BC	$t_{BC} \times \left(\frac{3500}{2.4}\right) = 5250$
E	3.6	0	BD	$t_{BD} \times \left(\frac{2000}{2.4}\right) = 2500$
			CE	$t_{CE} \times \left(\frac{-7000}{2.4}\right) = -8750$
			DE	$t_{DE} \times \left(\frac{-3000}{2.4}\right) = -3000$

Joint A,

$$t_{AD} (0 + 1.8) + t_{AB} (3.6) = 0$$

$$t_{AD} (-2.4) + t_{AB} (0) - 2000 = 0$$

$$t_{AD} = \frac{2000}{-2.4} = -\left(\frac{2000}{2.4}\right)$$

$$t_{AB} = -1.8 \frac{t_{AD}}{3.6} = -1.8 \times \frac{1}{3.6} \times \frac{-2000}{2.4} = \left(\frac{1000}{2.4}\right)$$

Joint D,

$$t_{AD} (-1.8) + t_{DB} (1.8) + t_{DE} (3.6) = 0$$

$$t_{AD} (2.4) + t_{DB} (2.4) + t_{DE} (0) = 0$$

$$t_{DB} = -t_{AD} = \left(\frac{2000}{2.4}\right)$$

$$-\frac{2000}{2.4} \times -1.8 + 1.8 \times \frac{2000}{2.4} + t_{DE} \times 3.6 = 0$$

$$t_{DE} = -\left(\frac{2000}{2.4}\right)$$

Joint B,

$$t_{BA} (-3.6) + t_{BD} (-1.8) + t_{BE} (1.8) + t_{BC} (3.6) = 0$$

$$t_{BA} (0) + t_{BD} (-2.4) + t_{BE} (-2.4) + t_{BC} (0) - 1000 = 0$$

$$t_{BE} + t_{BD} = -\frac{1000}{2.4}$$

$$t_{BE} = -\frac{1000}{2.4} - \frac{2000}{2.4} = -\left(\frac{3000}{2.4}\right)$$

$$\frac{1000}{2.4} \times -3.6 + \frac{2000}{2.4} \times -1.8 + \frac{-3000}{2.4} \times 1.8 + t_{BC} \times 3.6 = 0$$

$$\frac{1000}{2.4} (-3.6 - 3.6 - 5.4) = -3.6 t_{BC}$$

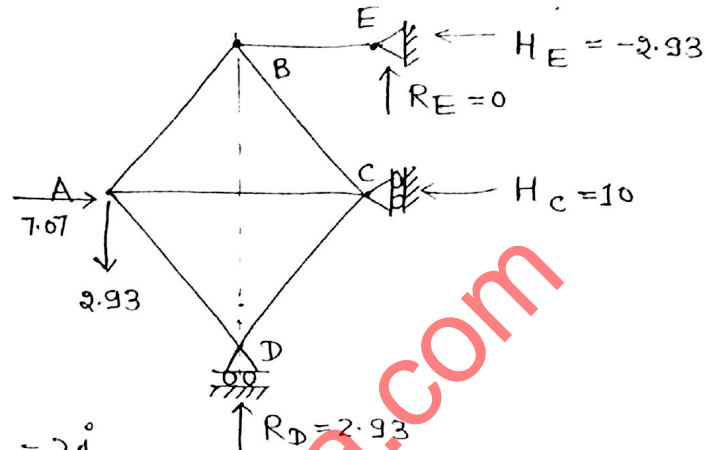
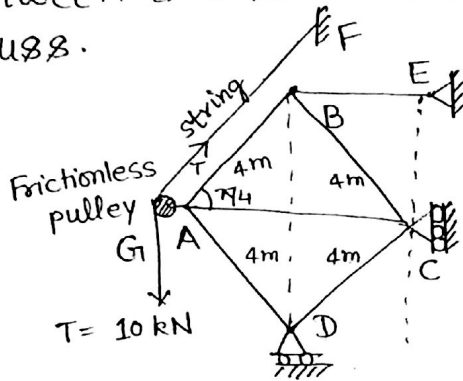
$$t_{BC} = \left(\frac{3500}{2.4}\right)$$

Joint C,

$$t_{CB} (-3.6) + t_{CE} (-1.8) = 0$$

$$t_{CE} = -2 t_{CB} = -\left(\frac{7000}{2.4}\right)$$

Question A square truss ABCD shown below carries a load of 10 kN attached to a string GF which is passing over a frictionless pulley as shown in figure. GF and AB are parallel. The truss is supported on rollers at C and D. A link BE is connected between B and E. Find forces in all members of truss.



$$D_s = m + r_e - 2j$$

$$= 6 + 4 - 2 \times 5 = 0$$

At Joint E,



$$\sum F_y = 0,$$

$$R_E = 0$$

$$R_D + R_E - 2.93 = 0$$

$$R_D = 2.93 \text{ kN}$$

$$\sum F_x = 0,$$

$$7.07 - H_C - H_E = 0$$

$$H_C + H_E = 7.07 \quad \text{--- (1)}$$

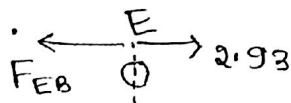
$$\sum M_C = 0,$$

$$R_D \times \frac{4}{\sqrt{2}} - 2.93 \times 4\sqrt{2} - H_E \times \frac{4}{\sqrt{2}} = 0$$

$$H_E = -2.93 \text{ kN}$$

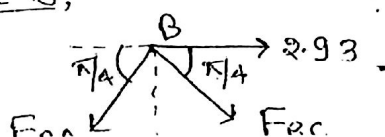
$$H_C = 10 \text{ kN}$$

Consider joint E,



$$F_{EB} = +2.93 \text{ kN}$$

Joint B,



$$F_{BC} \cos \frac{\pi}{4} + 2.93 - F_{BA} \cos \frac{\pi}{4} = 0$$

$$(F_{BC} + F_{BA}) \cos \frac{\pi}{4} = 0$$

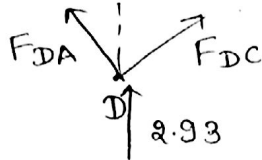
$$F_{BC} = -F_{BA}$$

$$-2 F_{BA} \cos \frac{\pi}{4} = -2.93$$

$$F_{BA} = \underline{2.07}$$

$$F_{BC} = \underline{-2.07}$$

Joint D,



$$F_{DC} \cos \frac{\pi}{4} + F_{DA} \cos \frac{\pi}{4} + 2.93 = 0$$

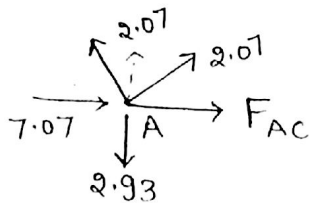
$$F_{DC} = F_{DA}$$

$$2 F_{DC} \cos \frac{\pi}{4} = -2.93$$

$$F_{DC} = \underline{-2.07}$$

$$F_{DA} = \underline{-2.07}$$

Joint A,

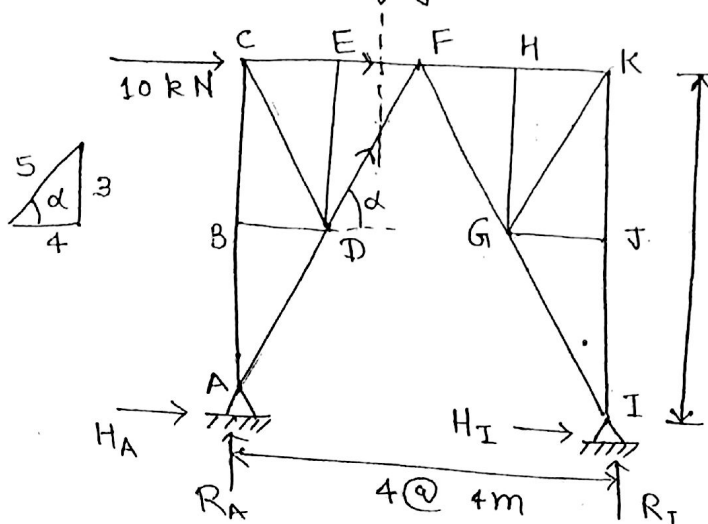


$$F_{AC} + 7.07 = 0$$

$$F_{AC} = \underline{-7.07}$$

Member	Force	Member	Force
AB	2.07	BE	+2.93
AC	-7.07	CD	-2.07
AD	-2.07		
BC	-2.07		

Question Find forces in members DF and EF for the truss shown in figure.



$$\sum F_x = 0,$$

$$R_A = -3.75 \text{ kN}$$

$$R_I = +3.75 \text{ kN}$$

$$H_A = H_I = 5 \text{ kN}$$

$$F_{EF} + 10 + H_A$$

$$+ F_{DF} \cos \alpha = 0$$

$$F_{EF} + F_{DF} \cos \alpha = -5$$

